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Amplified spontaneous emission IV. Beam divergence and spatial coherence

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Abstract. The origins of beam divergence in amplified spontaneous emission are discussed and theoretical models are developed to predict what their effect would be in real systems. The divergence of the beam is experimentally investigated for the 3·39 μm He–Ne and the 0·614 μm Ne systems and compared with the theories. Good agreement is achieved in pure neon for a theory which depends upon the parametric relationship between intensity and length of active medium. In the case of He–Ne although the theory gives the correct qualitative prediction, the absolute magnitude of the effect is wrong by a factor of about 1·7. Measurements of the degree of spatial coherence as a function of length and inversion density of the medium, and for the variation of spatial coherence across the tube for a given length and inversion density of medium, are presented for the neon system.

1. Introduction

In a recent series of papers (Peters and Allen 1971, Allen and Peters 1971a, Allen and Peters 1971b, to be referred to as I, II and III respectively) the onset condition for amplified spontaneous emission (ASE) to occur, the relationship between this condition and laser theory and the way in which the ASE intensity builds up and saturates as a function of length and diameter of the medium and of its degree of inversion, have all been derived. All relationships were confirmed experimentally using the 3·39 μm He–Ne, the 0·614 μm neon or the 0·337 μm nitrogen system.

In this paper the possible origins of beam divergence in a typical ASE system are discussed. These are diffraction effects at the exit aperture and the effect of the particular geometry of the column of atoms. In visible ASE systems (eg 0·540 μm and 0·614 μm neon systems) it is known that the latter is usually the predominant factor but for infrared systems (eg 3·39 μm He–Ne and 10·6 μm CO_2 systems), both appear to be important and in fact the former may become the predominant factor because the effects of diffraction increase with increasing wavelength. Theories are developed to account for these possible contributions to beam divergence, each of which can then be solved numerically using the parameters appropriate to a particular system.

Experimental results are presented for the 3·39 μm He–Ne system and the 0·614 μm neon system to enable a comparison of theories to be made. Many workers, for example, Leonard (1965), Rosenberger (1964) and Egorov and Plekhotkin (1969), have observed the 0·337 μm nitrogen system and 0·540 μm neon system and have found the divergence of the ASE to be given approximately by the ratio of geometric width to length d/L of the medium. They have not, however, indicated why this should be, nor presented any detailed measurements. Andronova *et al* (1968) looked specifically at the 3·39 μm transition in He–Ne and found the value of the beam divergence to be very much greater than d/L. They attributed this to tube reflections, and while this may be correct it is impossible to know for certain as the precise details of their system, and how the divergence was determined, are not known. It could be due to the effect of diffraction being the dominant mechanism.

Similarly no quantitative measurements have been recorded regarding the spatial coherence of the beam. Leonard and Zinky (1968) observed that fringes could be obtained in a typical two-beam interference experiment for all pinhole separations across the beam. Since fringes were always obtained they concluded that a high degree of spatial coherence existed right across the beam, but they did not measure the fringe visibility. Nor was any attempt made to determine how the spatial coherence was varying as a function of the system parameters. Experimental results are presented for the spatial coherence at different positions of the ASE beam as a function of tube length and inversion density for the 0·614 μm neon system and a discussion is offered as to their meaning.

2. Beam divergence

2.1. Theory

Consider a column of excited atoms of length L and width d, the stimulated emission from such a system will inevitably have a certain divergence. The degree of divergence will be decided by either (i) diffraction effects at the exit aperture of the column of atoms or (ii) the particular geometry of the column of gas. To decide which of these is applicable to any particular ASE system, and in what way, the magnitude of the divergence for each effect must be computed.

2.1.1. The effect of diffraction. Koppelmann (1969) considered a general externally excited interferometer consisting of two parallel plates and determined the angular distribution of emitted radiation taking into account both multiple beam interference and diffraction effects. Consider a plane wave of amplitude A_0 and wavelength λ incident on infinite strip mirrors with reflection coefficient R, transmission coefficient T, width d and separation b, at an angle θ_0 to the axis of the plates. He showed that the complex amplitude distribution A_θ of the output as a function of angle from the plate axis θ was given by

$$A_\theta = \frac{1}{2}A_0 \sum_{m=0}^{\infty} \left(\frac{T\exp(-\gamma_m b)}{1-R\exp(-2\gamma_m b)} \frac{\sin\{\phi_0 - \frac{1}{2}(m+1)\pi\}}{\phi_0 - \frac{1}{2}(m+1)\pi} \times \frac{\sin\{\phi - \frac{1}{2}(m+1)\pi\}}{\phi - \frac{1}{2}(m+1)\pi} \frac{(m+1)^2\pi^2}{\{\phi_0 + \frac{1}{2}(m+1)\pi\}\{\phi + \frac{1}{2}(m+1)\pi\}} \right) \tag{1}$$

where

$$\phi_0 = \frac{\pi d}{\lambda}\sin\theta_0 \qquad \phi = \frac{\pi d}{\lambda}\sin\theta$$

$$\gamma_m = \alpha_m + \mathrm{i}\beta_m \qquad \alpha_m = \frac{0{\cdot}512(b\lambda^3)^{1/2}(m+1)^2}{d^3}$$

$$\beta_m = \frac{2\pi}{\lambda}\left(1 - \frac{(m+1)^2\lambda^2}{(2d)^2}\right)^{1/2}$$

and the summation is over all modes of the cavity.

Allowing $R \to 0$ and $T \to 1$ suppresses the multiple beam reflections and leaves only diffraction effects. So effectively there is a column of length b and width d externally excited at an angle θ_0 by a plane wave and the formulation determines the spatial distribution of emitted radiation from the other end due to diffraction effects alone. Even allowing for the fact that the infinite strip reflector is only going to permit a discussion of beam divergence in the plane perpendicular to the mirror axis, one might expect this approach to be only approximately correct. It clearly does not correspond exactly to the situation that exists in a typical ASE system because (i) the latter is not externally excited, but the wave builds up within the column, (ii) the column contains an active medium rather than a passive one and (iii) the ASE wave is not in general plane. The justification for ignoring the first objection is that if there is a tube of length L and critical length L_c, then at the point in the tube $x = L_c$ an ASE wave has built up due to stimulated emission and it can now be considered that this is incident upon the remaining length of tube $(L-L_c)$. That is, b should be taken as $(L-L_c)$ in the above equation rather than as L. The second objection implies that stimulated emission is going to cause the amplitude of the wave to increase in the distance $(L-L_c)$. The fact that this is so, however, only affects, to a first order, the absolute amplitude of the emitted radiation and not its angular distribution. Also spontaneous emission has only been considered at the origin of the tube $x = 0$ producing a wave at $x = L_c$, whereas all spontaneous emission from $x = 0$ up to $x = (L-L_c)$ is going to produce a similar wave, only it will act on a smaller length of medium. However, it transpires that as the length of medium upon which the wave is incident is decreased, the angular distribution of the radiation is compressed. So the maximum spread of the radiation is going to be determined by that incident wave produced at $x = L_c$ from spontaneous emission at $x = 0$. This light is also the most intense, since it sees a maximum amplifying path. The other waves resulting from spontaneous emission in the region $x = 0$ to $x = (L-L_c)$ are going to modify the distribution to some extent (it will tend to make the distribution slightly narrower), but to a first order ignoring this objection does not seem unreasonable.

Bearing in mind that these objections are being ignored and the fact that a plane wave is assumed, it is now possible to write down the angular distribution of the complex amplitude of ASE due to diffraction as

$$A_\theta = A_0 \sum_{m=0}^{\infty} (\mathrm{e}^{-\gamma_m b}) \frac{\cos\{(m+1)\pi - \phi\} - \cos\phi}{\phi^2 - \frac{1}{4}(m+1)^2\pi^2} \qquad (2)$$

where $R = 0$, $T = 1$ and $\theta_0 = 0$, that is, the radiation is at normal incidence.

Equation (2) can now be solved numerically for a given value of d, b and λ to obtain the complex amplitude $A_\theta = B_\theta + \mathrm{i}C_\theta$ as a function of angle θ from the column axis. Finally the intensity distribution may be determined by taking the square modulus of A_θ. Thus the beam width at half intensity may be computed as a function of tube length L and critical length L_c.

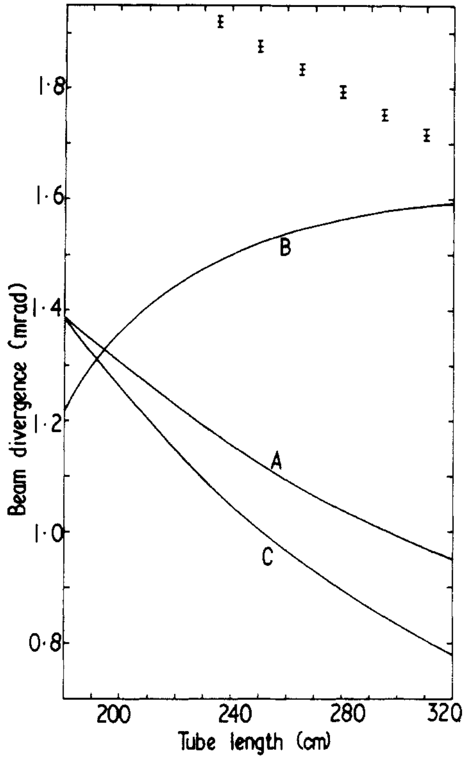
2.1.2. The effect of geometry. Consider a tube of length L and width d containing an active medium with a critical length L_c. It has been shown in I that a certain critical length has to exist before ASE occurs, and that only spontaneous emission in the region $x = 0$ to $x = (L - L_c)$ can act as a source for ASE (see III). Spontaneous emission originating along the tube axis at $x = 0$ will give rise to an ASE output with divergence given by d/L whilst spontaneous emission along the tube axis at $x = (L - L_c)$ will give rise to an ASE output with divergence given by d/L_c, provided $L_c \gg d$. However, as far as intensity is concerned the contributions are not equal because that spontaneous emission originating at $x = 0$ has traversed a longer amplifying path than that at $x = (L - L_c)$. So it appears that to obtain the spatial intensity distribution of the ASE output, for a tube of length L, it is necessary to sum the contributions due to all elements in the region $x = 0$ to $x = (L - L_c)$, for light emitted from any point in the tube cross section, taking into account both the ASE intensity of the emitted radiation due to spontaneous emission from these elements and also the angle into which the spontaneous radiation can be emitted and yet appear at the exit aperture of the tube. This has been done (Peters 1971) and leads to what is described in the rest of this paper as an ASE-geometric theory.

The effects of single or multiple reflections from the tube walls on the divergence of the beam ought to be considered. This results in radiation still being transmitted throughout the entire length of the tube even though it is radiated into a larger solid angle than previously defined. Thus an increase in beam divergence would be expected although the precise magnitude would be difficult to predict since it depends upon the nature of the wall reflectivity.

2.2. Comparison of beam divergence theory with experiment

The experimental arrangements for both the 3·39 μm He–Ne system and 0·614 μm neon system are exactly the same as those described in the previous papers. In both cases the tube bore was 2·5 mm diameter. The beam divergence was determined experimentally by measuring the spatial distribution of radiation at various distances from the tube end, as a function of both tube length and inversion density, and then computing the divergence from the width of the distribution at the half intensity points. For the He–Ne system this was accomplished by manually tracking a detector, with a pinhole aperture, across the beam. In the neon system the ASE output for a single pulse was photographed and the developed negative scanned with a microdensitometer to obtain a corresponding transmission trace. The width of the ASE distribution at the half intensity point is then proportional to the width of the densitometer trace at the point where the film transmission is twice the minimum value T_0. To ensure that both T_0 and $2T_0$ lay on the linear region of the film characteristic curve, as length and inversion were varied, two polaroid sheets were used with the angle between their axes suitably adjusted for each measurement to reduce the intensity of the light striking the film to an appropriate level.

Figures 1 and 2 show the variation of beam divergence according to the Koppelmann diffraction theory, the ASE-geometric theory and, for completeness, the simple d/L geometric ratio, together with the experimental points for the 3·39 μm He–Ne system as a function of tube length (for inversion density = 15·8 units) and of inversion density (for tube length = 310 cm) respectively. Note in figure 1 that diffraction effects cause the divergence to increase as the tube length increases. Geometric considerations would cause it to decrease, and this seems to be in agreement with experiment although the experimental points are high by a factor of about 1·7. Figure 2 shows again that the experimental results are high but of the two geometrically limited curves they are qualitatively in agreement with the ASE-geometric approach.



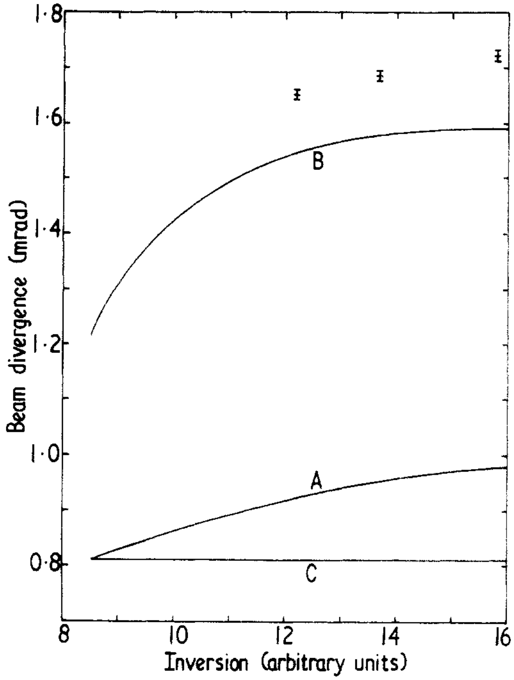


Figure 1. Experimental points (ɪ) and theoretical curves for the ASE-geometric (A), Koppelmann diffraction (B) and simple d/L (C) theories for the variation of beam divergence with tube length, using an inversion density of 15·8 units and a tube bore of 2·5 mm, for the 3·39 μm He–Ne ASE transition.

Figure 2. Experimental points (ɪ) and theoretical curves for the ASE-geometric (A), Koppelmann diffraction (B) and simple d/L (C) theories for the variation of beam divergence with inversion density, using a tube of length 310 cm, and a bore 2·5 mm for the 3·39 μm He–Ne ASE transition.

Figures 3 and 4 show the corresponding results in neon with an inversion density of 100 units and a tube length of 110 cm respectively. The diffraction value for the beam divergence according to the Koppelmann theory for a tube of length 110 cm, a bore of 2·5 mm and an inversion of 100 units is 0·232 mrad—some 1·5 orders of magnitude down on the geometric value. It is interesting to note that whereas in the He–Ne system the experimental points are rather high compared to the ASE theoretical curve, the results in neon agree to within the experimental error ($\simeq 5\%$). Possibly this is because of the effect of reflections in the tube. In each system the material used was pyrex, and at grazing incidence the coefficient of reflection is always approximately unity. Ordinary drawn pyrex tubing was used for the neon system and its inner surface was appreciably less even than the precision bore tubing used at 3·39 μm. Possibly this has some significance in accounting for the fact that although the qualitative prediction for 3·39 um is correct, it is numerically wrong by a factor of about 1·7. Certainly reflections would tend to increase the beam divergence rather than lessen it.

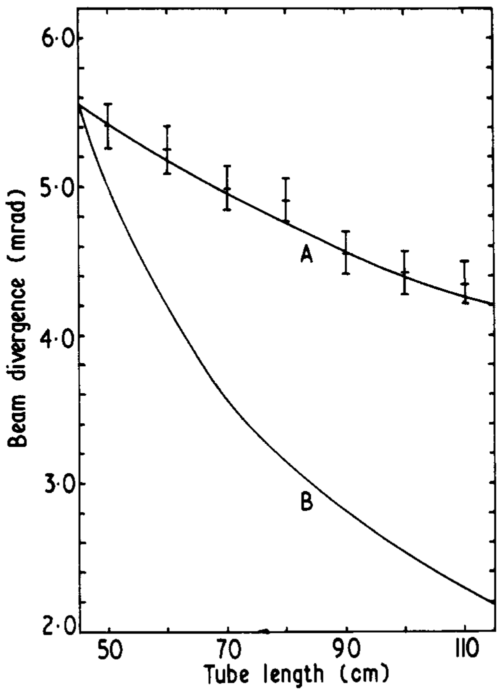


Figure 3. Experimental points (I) and theoretical curves for the ASE-geometric (A) and simple d/L (B) theories for the variation of beam divergence with tube length, using an inversion density of 100 units and a tube bore of 2·5 mm, for the 0·614 μm neon ASE transition. The diffraction theory is not shown since it is 1·5 orders of magnitude smaller.

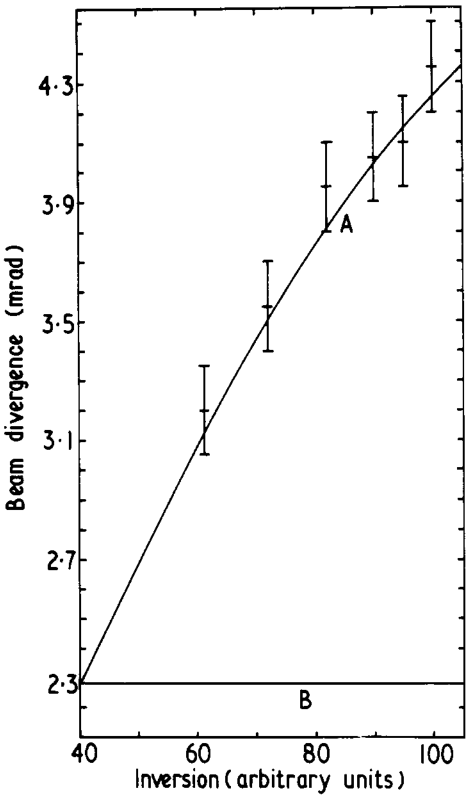


Figure 4. Experimental points (I) and theoretical curves for the ASE-geometric (A) and simple d/L (B) theories for the variation of beam divergence with inversion density, using a tube of length 110 cm and bore 2·5 mm, for the 0·614 μm neon ASE transition. The diffraction theory is not shown since it is 1·5 orders of magnitude smaller.

3. Spatial coherence

3.1. Experimental results

The spatial coherence of the 0·614 μm neon system was investigated by observing the visibility of the fringes produced in a typical two-beam interference experiment employing two pinholes. The fringes so produced were photographically recorded and a corresponding transmission trace of the developed film obtained with the microdensitometer as for the beam divergence experiment. Here it was not always possible to arrange for the maximum and minimum transmissions of the film to fall on the linear region of the characteristic curve for the film. It was necessary to convert transmission readings to intensity readings via the characteristic curve with potentially large errors occurring for transmissions off the linear region.

Figure 5 shows how the spatial coherence varies when the separation of the pinholes causing the fringes, symmetrically displaced about the tube axis, is varied for a tube length

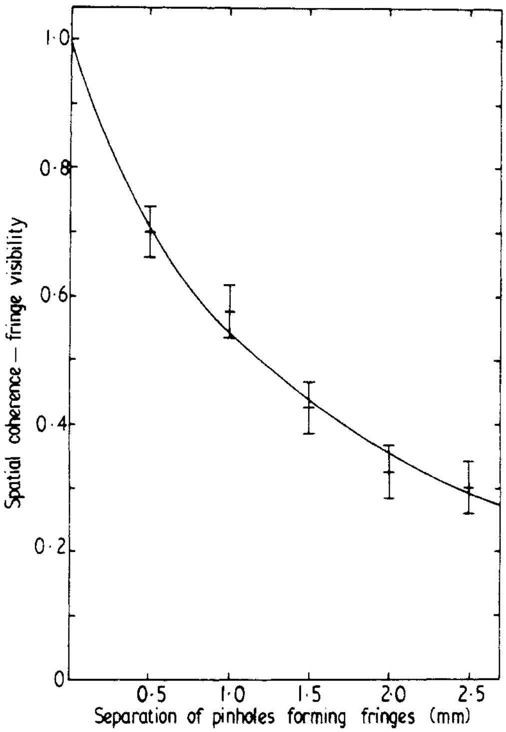


Figure 5. Plot of the variation of spatial coherence with the separation of the pinholes causing the fringes symmetrically displaced about the axis, using a tube of length 100 cm and an inversion density of 90 units, for the 0·614 μm neon system.

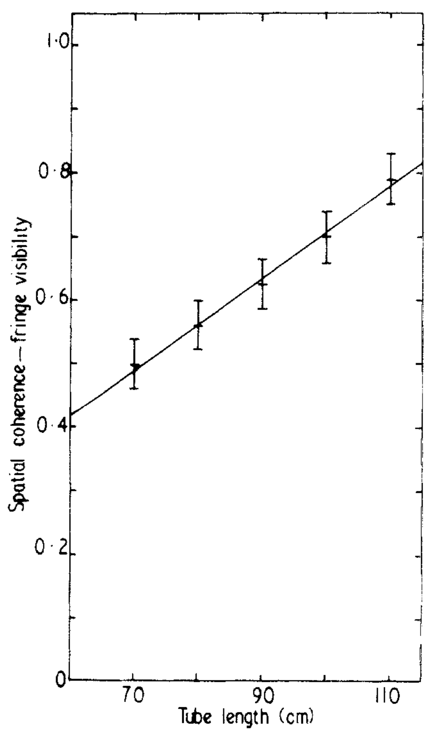


Figure 6. Plot of the variation of spatial coherence with tube length, using an inversion density of 90 units and a pinhole separation of 0·5 mm, for the 0·614 μm neon system. The pinholes are symmetrically displaced about the axis.

of 100 cm and an inversion density of 90 units. Thus this indicates how the spatial coherence is varying across the ASE beam. Figure 6 is a plot of spatial coherence against tube length for an inversion density of 90 units and a pinhole separation of 0·5 mm, and indicates that the coherence increases linearly with length. If the coherence was exclusively a property of the radiation process itself then one would perhaps expect the increase to be related to the form of the intensity against length curves of III. For completeness figure 7 shows the variation of spatial coherence with inversion density for a tube length of 100 cm and a pinhole separation of 0·5 mm.

An interesting phenomenon occurs when fringes in a two-beam interference experiment of the type previously described, are photographed for a single light pulse, which has a very short time duration and is temporally coherent over the whole pulse. Martienssen and Spiller (1964) have shown that interference fringes under these conditions can be obtained, even if the light is not spatially coherent. However, if observations are made over a number of pulses then, because the maxima and minima of the fringes change their position statistically, and fringes are destroyed. Glas (1970) observed exposures for 50 pulses and 2000 pulses in the 0·614 μm neon system and found that fringes looked the same. So the fringe formation in the Ne ASE system is shown to be due to spatial rather than temporal properties.

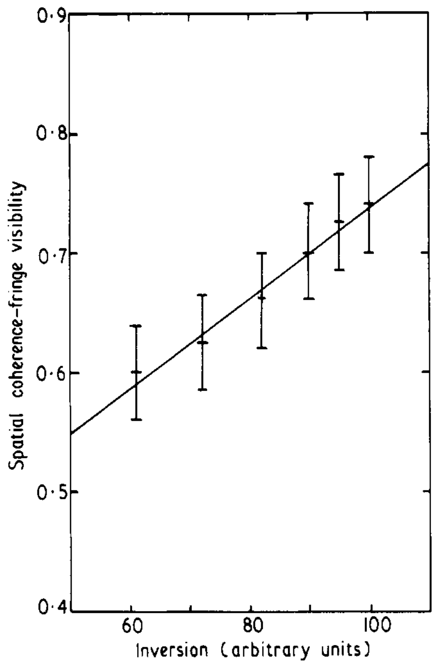


Figure 7. Plot of the variation of spatial coherence with inversion density, using a tube length of 100 cm and a pinhole separation of 0·5 mm, for the 0·614 µm neon system. The pinholes are symmetrically displaced about the axis.

4. Conclusions

Using the results reported in III and by use of simple geometric optics we have been able to account for the observed divergence of the ASE beam, provided reflections from the walls of the tube containing the active medium are not important. Further we have tentatively shown that the spatial coherence of the 0·614 µm ASE beam is more a property of the geometry of the tube containing the active medium rather than of the medium itself or of the radiation process. To investigate this further a careful study is necessary to find the exact dependence of spatial coherence on the tube dimensions and reflectivity of the tube wall, in the presence and absence of resonant atoms. A preliminary study has begun in this laboratory (see Allen *et al* 1971).

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